

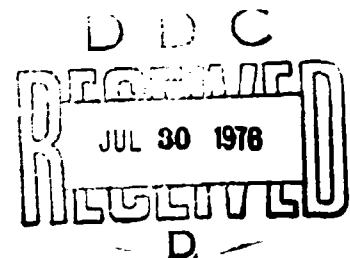
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PHOTOELECTRON NOISE LIMITATIONS IN HIGH
PERFORMANCE IMAGING SENSORS



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INTRODUCTION

In the conventional signal-to-noise ratio analysis (1) of image forming sensors, the imaged area is broken into fictional "resolution elements" and the noise calculated as the square root of the total number of counts or events, n , falling within a single element. Since the signal is proportional to the total number of counts within the element, the signal-to-noise ratio is $n^{1/2}$. By postulating a minimum required, or threshold, signal-to-noise ratio, these calculations have led to expressions (2) for the limiting resolution of the device as a function of incident light level. The effects of a finite aperture have been considered by Schade (3) and more recently by Rosell (4). Schade's analysis led him to define the noise equivalent sampling area as the reciprocal of the noise equivalent passband. The concept of a noise equivalent sampling area has been used by Rosell to calculate the display signal-to-noise ratio and limiting resolution of a variety of television camera tubes. Analyses of this type have been extensively used to calculate the performance of military television systems.

In the following analysis some of the ambiguities of the Rosell-Schade approach are resolved. The apriori assumption of the existence of fictitious resolution elements is eliminated and reintroduced only after considerable development. Two equivalent square sampling apertures or areas are defined: a signal sampling aperture, x_s , and a noise sampling aperture, x_n . It will be shown that the noise equivalent sampling area of Schade is given by $x^* \equiv 1/N_e = x_s^2/x_n$; the proper interpretation of x^* is thus as an equivalent aperture that weights both signal and noise, not noise alone.

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The theoretical development is in the language of television or raster scan type sensors. This is not essential and with minor modification can be applied to other imaging sensors such as direct view intensifiers and discrete solid state arrays.

THEORY

General

For convenience and brevity of notation, a one dimensional imaging system is assumed. With certain simplifying assumptions, the extension to two dimensions is straightforward and will be indicated later. The input and output of a linear, spatially invariant system are related by (5)

$$g_o(x) = \int_{-\infty}^{\infty} h(x-x') g_i(x') dx' \quad (1)$$

where g_i and g_o are the input and output respectively and $h(x-x')$ is the impulse or point response of the system. That is, if $g_i(x') = \delta(x'-x'_0)$, then $g_o(x) = h(x-x'_0)$. It is assumed that the system has some characteristic integration time τ such that during the time interval $t, t + \tau$, K photoelectrons are emitted at positions x'_1, x'_2, \dots, x'_K . The input is thus $g_i(x') = \sum \delta(x'-x'_j)$ and the output image at time $t + \tau$ is $g_o(x) = \tau h(x-x'_j)$. In general, the K emission positions and the value of K itself form a set of $K + 1$ independent random variables so that the average output, $E[g_o(x)]$, is given by (6):

$$E[g_o(x)] = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{i=1}^K h(x-x'_i) p(x'_1, \dots, x'_K) p(K) dx'_1 \dots dx'_K dK \quad (2)$$

where $p(x'_1 \dots x'_K) dx'_1 \dots dx'_K$ is the probability that the K electrons are emitted at positions x'_1, \dots, x'_K and $p(K)dK$ is the probability that K electrons are emitted during the time interval. To evaluate the output noise, the following expression for the variance is required:

$$\begin{aligned} \sigma^2 &\equiv E[|g_o(x) - E[g_o(x)]|^2] \\ &= \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \sum_{i,j=1}^K h(x-x'_i) h(x-x'_j) p(x'_1, \dots, x'_K) \\ &\quad p(K) dx'_1 \dots dx'_K dK - E^2[g_o(x)] \end{aligned} \quad (3)$$

The signal-to-noise ratio is defined as $S/N = E[g_o(x)]/\sigma$ and, in general, is a function of position. Equations (2) and (3) will be evaluated for four cases of special interest.

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Uniform Background

For a uniformly illuminated background, the photoelectrons are emitted at purely random positions (Poisson process) so that

$$p(x'_1, x'_2, \dots, x'_K) dx'_1 \cdots dx'_K = \frac{e^{-K}}{K!} \prod_{i=1}^K dx'_i \quad -\frac{w}{2} \leq x'_i \leq \frac{w}{2} \quad (4)$$

$$= 0, \quad |x'_i| > w/2$$

where w is the width of the photocathode. Furthermore, $p(K)dK$ is given by

$$p(K)dK = \frac{e^{-\bar{K}}}{K!} \bar{K}^K dK \quad (5)$$

where \bar{K} is the average number of photoelectrons emitted during the time interval. Substitution of Equation (4) into (2) gives

$$E[g_0(x)] = \int_0^\infty p(K)dK \left[\sum_{i=1}^K \frac{1}{w} \int_{-w/2}^{w/2} h(x-x'_i) dx'_i \right] \quad (6)$$

Neglecting edge effects, the expression in brackets can be written as

$$\sum_{i=1}^K \frac{1}{w} \int_{-w/2}^{w/2} h(x-x'_i) dx'_i = \sum_{i=1}^K \frac{1}{w} \int_{-\infty}^{\infty} h(x-x'_i) dx'_i = \frac{K}{w} \int_{-\infty}^{\infty} h(x) dx \quad (7)$$

Then Equation (6) becomes

$$E[g_0(x)] = v_B \int_{-\infty}^{\infty} h(x) dx \quad (8)$$

where $v_B = \bar{K}/w$ is the average number of photoelectrons per unit distance emitted from the photocathode during the time interval. A similar calculation for the variance gives

$$\sigma^2 = v_B \int_{-\infty}^{\infty} h^2(x) dx \quad (9)$$

so that the signal-to-noise ratio is

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$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty} h(x) dx}{[\int_{-\infty}^{\infty} h^2(x) dx]^{1/2}} \quad (10)$$

and is independent of x .

Isolated Point Source

Consider a small but finite "point" source of photoelectrons centered at $x' = x'_0$ with width a . Then

$$p(x'_1, \dots, x'_K) dx'_1 \dots dx'_K = \frac{1}{a^K} \prod_{i=1}^K dx'_i; \quad x'_0 - \frac{a}{2} < x'_i < x'_0 + \frac{a}{2} \quad (11)$$

= 0 otherwise

With this substitution Equation (2) becomes

$$E [g_0(x)] = \int_{-\infty}^{\infty} p(K) dK \left[\sum_{i=1}^K \frac{1}{a} \int_{x'_0 - a/2}^{x'_0 + a/2} h(x-x'_i) dx'_i \right] \quad (12)$$

Since a is small,

$$\frac{1}{a} \int_{x'_0 - a/2}^{x'_0 + a/2} h(x-x'_i) dx'_i \approx h(x-x'_0)$$

and

$$E [g_0(x)] = \int_{-\infty}^{\infty} K p(K) dK h(x-x'_0) \quad (13)$$

$\equiv \bar{K} h(x-x'_0)$

A similar calculation for the variance gives

$$\sigma^2 = \bar{K} h^2(x-x'_0) \quad (14)$$

and the signal-to-noise ratio is

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$$S/N = \bar{K}^{1/2}, \quad (15)$$

an easily anticipated result.

Point Source in a Uniform Background

The signal-to-noise ratio can be calculated using the same methods as before but a simpler procedure is to use system linearity and superimpose the previous results. It is found that

$$E \{g_o(x)\} = v_B \int_{-\infty}^{\infty} h(x) dx + \bar{K} h(x-x'_0) \quad (16)$$

and

$$\sigma^2 = v_B \int_{-\infty}^{\infty} h^2(x) dx + \bar{K} h^2(x-x'_0) \quad (17)$$

The signal-to-noise ratio is then

$$\frac{S}{N} = \frac{v_B \int_{-\infty}^{\infty} h(x) dx + \bar{K} h(x-x'_0)}{[v_B \int_{-\infty}^{\infty} h^2(x) dx + \bar{K} h^2(x-x'_0)]^{1/2}} \quad (18)$$

In Equations 16 - 18, \bar{K} is the average number of emitted photoelectrons due to the point source alone and v_B the average density of emitted photoelectrons due to background alone.

Periodic Input

A case of interest, particularly in the laboratory evaluation procedure, is the case of a periodic input for which the average brightness is $B(x') = B(o) [1 + c (2-c)^{-1} \cos \omega x']$. The quantity $B(o)$ is the maximum brightness and c is the contrast. The probability density function is found to be

$$\begin{aligned} p(x'_1, \dots, x'_K) dx'_1 \dots dx'_K &= w^{-K} \left(1 + \frac{c}{2-c} \operatorname{sinc} \omega w\right)_{i=1}^{K-1} \left(1 + \frac{c}{2-c} \cos \omega x'_1\right) dx'_1; \\ &- \frac{w}{2} \leq x'_1 \leq w/2 \\ &= 0, \quad |x'_1| > w/2 \end{aligned} \quad (19)$$

After straightforward but tedious calculation,

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$$E [g_o(x)] = v_p \left[\int_{-\infty}^{\infty} h(x) dx + \frac{c}{2-c} \cos \omega x \int_{-\infty}^{\infty} h(x') \cos \omega x' dx' \right] \quad (20)$$

$$\sigma^2 = v_p \left[\int_{-\infty}^{\infty} h^2(x) dx + \frac{c}{2-c} \cos \omega x \int_{-\infty}^{\infty} h^2(x') \cos \omega x' dx' \right] \quad (21)$$

where v_p is the peak photoelectron density given by

$$v_p \equiv \frac{\bar{K}}{W} \left(1 + \frac{c}{2-c} \operatorname{sinc} \frac{\omega W}{2} \right)^{-1}$$

From Equations 20 and 21, the signal-to-noise ratio is

$$\frac{S}{N} = \frac{v_p \left[\int_{-\infty}^{\infty} h(x) dx + \frac{c}{2-c} \cos \omega x \int_{-\infty}^{\infty} h(x') \cos \omega x' dx' \right]}{\left[\int_{-\infty}^{\infty} h^2(x) dx + \frac{c}{2-c} \cos \omega x \int_{-\infty}^{\infty} h^2(x') \cos \omega x' dx' \right]^{1/2}} \quad (22)$$

Definition of Averaging Distances

Equations 10, 15, 18 and 22 are expressions for the signal-to-noise ratio in terms of various integrals (or averages) over the point response function. Although not essential, it is convenient to rewrite these equations in terms of averaging distances or apertures so that a simple "resolution element" analysis can be used. Two such sampling distances are defined: the signal sampling distance, x_s , is defined as

$$h(o) x_s = \int_{-\infty}^{\infty} h(x) dx$$

and the noise sampling distance, x_n , as

$$h^2(o) x_n = \int_{-\infty}^{\infty} h^2(x) dx$$

The signal-to-noise ratio for a point source on a uniform background (Equation 18) can then be rewritten as

$$\frac{S}{N} = \frac{v_B h(o) x_s + \bar{K} h(x-x_o')}{[v_B h^2(o) x_n + \bar{K} h^2(x-x_o')]^{1/2}} \quad (23)$$

and at the peak of the point source, $x = x_o'$,

$$\frac{S}{N} = \frac{v_B x_s + \bar{K}}{[v_B x_n + \bar{K}]^{1/2}} \quad (24)$$

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Equation 24 has the desired form of a ratio of signal counts to noise counts. For $\bar{K} = 0$ (background alone), Equation 24 becomes

$$\frac{S}{N} = (v_B x^*)^{1/2} \quad (25)$$

where $x^* \equiv x_s^2 / x_n$. It is a straightforward matter to show that $x^* = 1/N_e$ and that N_e is the noise equivalent passband of Schade. Thus for a uniform background, a single averaging distance is sufficient to calculate the signal-to-noise ratio. However, when considering a point source and a background, a two parameter description is required. It is concluded that both signal and noise averaging distances are required and that the noise averaging distance of Schade is actually a combination of signal and noise averaging distances.

The signal-to-noise ratio for a periodic input, Equation 22, can be rewritten as

$$\frac{S}{N} = v_p^{1/2} \frac{(x_s + \frac{c}{2-c} \cos \omega x \mathcal{F}\{f(x)\})}{[x_n + \frac{c}{2-c} \cos \omega x \mathcal{F}\{f^2(x)\}]^{1/2}} \quad (26)$$

where $f(x) \equiv h(x)/h(0)$ is the normalized impulse response and \mathcal{F} implies a Fourier transform.

Extension to Two Dimensions

If the system is spatially invariant in both dimensions so that $h(x, x'; y, y') = h(x-x'; y-y')$, the calculation proceeds along the lines of the one dimensional model. One obtains the following modifications:

$$x_s \rightarrow a_s = \frac{1}{h(0,0)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) dx dy$$

$$x_n \rightarrow a = \frac{1}{h^2(0,0)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h^2(x,y) dx dy$$

$v_B \rightarrow$ average area density of emitted photoelectrons

Although in general a raster scan system is spatially variant in the direction perpendicular to the scan direction, the data of a following sections shows that SIT or EBS type television tubes closely approximate a continuous scan system for scan line densities in excess of about 40 scan lines per mm of raster height.

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Some Useful Results

(1) The MTF and impulse response are Fourier transform pairs so that

$$h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega x} d\omega$$

and

$$H(\omega) = \int_{-\infty}^{\infty} h(x) e^{-i\omega x} dx$$

If the MTF is normalized at zero spatial frequency, then $\int_{-\infty}^{\infty} h(x) dx = 1$ and $x_s = h^{-1}(0)$. Therefore

$$x_s = 2\pi / \int_{-\infty}^{\infty} H(\omega) d\omega$$

Using Parseval's theorem, it can be shown that

$$x_n = \frac{2\pi \int_{-\infty}^{\infty} |H(\omega)|^2 d\omega}{[\int_{-\infty}^{\infty} H(\omega) d\omega]^2}$$

Thus if the MTF is known the signal and noise sampling distances can be calculated. Analogous results can be derived for a two dimensional system.

(2) A commonly observed impulse response is the Gaussian for which

$$h(x, x'; y, y') = (2\pi\sigma_x\sigma_y)^{-1} e^{-\frac{(x-x')^2}{2\sigma_x^2}} e^{-\frac{(y-y')^2}{2\sigma_y^2}}$$

then

$$a_s = 2\pi\sigma_x\sigma_y$$

$$a_n = \pi\sigma_x\sigma_y$$

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and

$$a_n = a_s/2$$

Thus the noise sampling area is half as large as the signal sampling area for a Gaussian impulse response.

(3) The signal-to-noise ratio for a periodic input and Gaussian impulse response is found to be

$$\frac{S}{N} = (v_p a^*)^{1/2} \frac{1 + \frac{c}{2-c} \cos \omega x e^{-\sigma_x^2 \omega^2 / 2}}{(1 + \frac{c}{2-c} \cos \omega x e^{-\sigma_x^2 \omega^2 / 4})^{1/2}} \quad (27)$$

Equation 27 can be used to derive an expression for the sensor limiting resolution by redefining the signal to be the peak-to-peak signal and averaging the spatially dependent noise over the photocathode area. Then

$$\frac{S}{N} = (v_p a^*)^{1/2} \frac{2c}{2-c} e^{-\sigma_x^2 \omega^2 / 2} \quad (28)$$

if $\omega w \gg 1$. Assuming that the limiting resolution, ω_l , occurs at some threshold signal-to-noise ratio k , then

$$\omega_l = \frac{\sqrt{2}}{\sigma_x} \left\{ \ln \frac{2c (v_p a^*)^{1/2}}{(2-c)k} \right\}^{1/2} \quad (29)$$

Up to this point only photoelectron noise has been considered. In the likely event that an internal noise source contributes an effective noise count n_p , it can be added in quadrature with the photoelectron noise. For example, Equation 29 would be modified to read as

$$\omega_l = \frac{\sqrt{2}}{\sigma_x} \left\{ \ln \frac{2c v_p a_s}{(2-c)k(v_p a_n + n_p)} \right\}^{1/2} \quad (30)$$

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EXPERIMENTAL

Measurement Technique

All uniform background data were taken using the US Army Night Vision Laboratory Television Test Set. This extremely flexible system permits a wide range of measurements under non-standard conditions. For example, line and frame rates are variable from 100-2000 lines/frame and from 1-60 frames/sec. An on-line Varian 6201 minicomputer and a Computer Lab Model 615 A/D converter are used for real time data collection and processing. Video is digitized into 6 bit brightness words at a 15 MHz sampling rate. A buffer memory accepts up to 2048 words which are then clocked into the computer at a 200 KHz rate. Thus up to 2048 points in a single frame of video can be sampled and fed to the computer. The sampled points can be adjacent points on a single line or selected segments of successive lines. The same points can be sampled over successive frames for a multi-frame average. The photoelectron noise is measured by calculating the standard deviation of the collected data and correcting for preamplifier noise.

Experimental Results

The photoelectron noise due to a uniform background is shown in Figure 1. These data were taken at 750, 525, and 300 scan lines per frame on a Westinghouse 31841G EBS tubes. This tubes has a 40 mm S-20 photocathode, a 25 mm deep-etched metal cap silicon array target, and a target gain of 1800 at 10 KV acceleration potential. Values of $a^* = a_s / a_n$ calculated from the data of Figure 1 are plotted in Figure 2 as a function of scan line density. The horizontal line is the expected a^* value for a symmetrical, continuous sampling system as calculated from measured MTF data. The sloping line is the expected value for a purely discrete sampler in the y direction and a continuous sampler in the x direction. The σ_x value is determined from horizontal MTF measurements. It is seen that the data approach a continuous sampling model for scan line densities in excess of approximately 700 scan lines/frame.

Figure 3 is a plot of photoelectron noise current versus signal current for a point source and a uniform background taken separately. These data were taken on an RCA SIT Tube Type 4826. This tube has a 25 mm photocathode, an 18 mm diode array target, and a target gain of 700 at 6 KV. Assuming a symmetrical, continuous scan model and using measured measured MTF data, the calculated value of a^* is $2.4 \times 10^{-3} \text{ mm}^2$. The measured value from Figure 3 is $2.3 \times 10^{-3} \text{ mm}^2$. It thus appears that 525 scan lines on an 18 mm target are sufficient for an

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WESTINGHOUSE TYPE 31841G
EBS TUBE
TARGET GAIN = 1800
VIDEO BANDWIDTH = 8 MHZ
NON-INTERLACED
30 FRAMES/SEC
○ 750 SCAN LINES
△ 525 SCAN LINES
□ 300 SCAN LINES

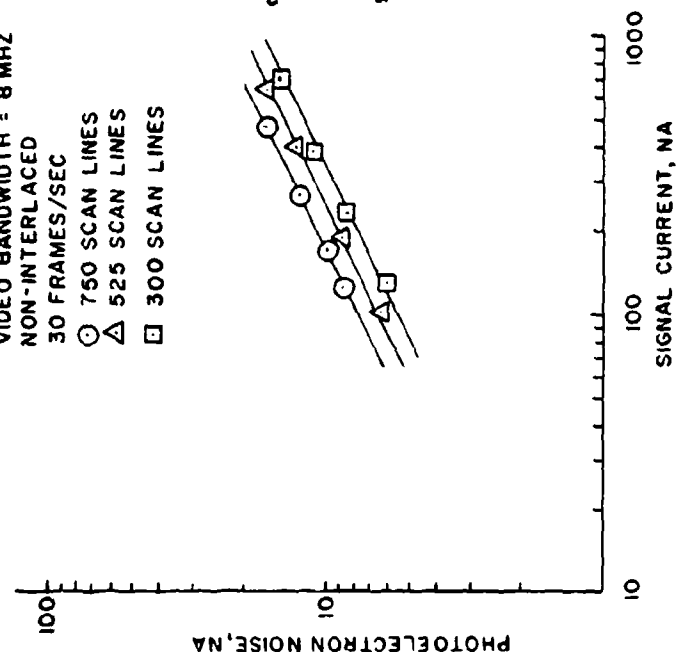


FIG. 1 PHOTOELECTRON NOISE,
UNIFORM BACKGROUND

WESTINGHOUSE TYPE 31841G
EBS TUBE
TARGET GAIN = 1800
VIDEO BANDWIDTH = 8 MHZ
NON-INTERLACED
30 FRAMES/SEC

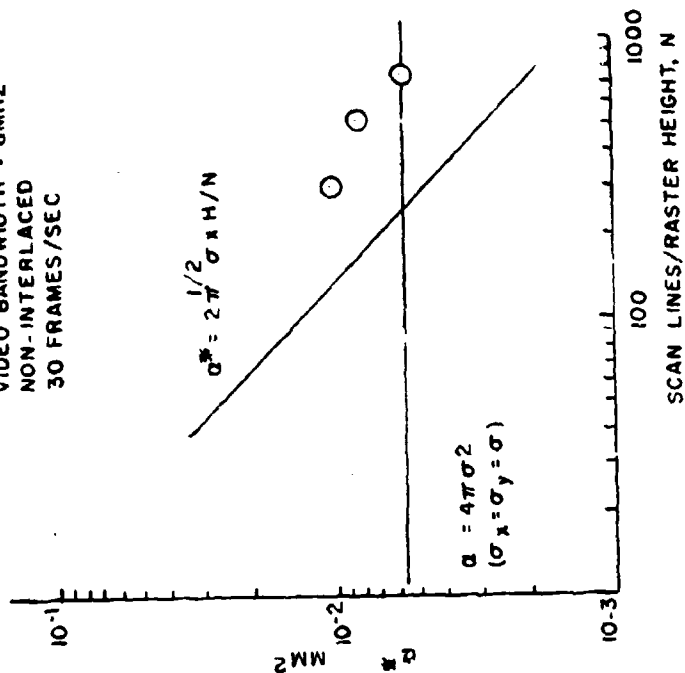


FIG. 2 SAMPLING AREA AS A FUNCTION
OF SCAN LINE DENSITY

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RCA TYPE 4826 SIT TUBE
TARGET GAIN = 700
525 SCAN LINES
30 FRAMES/SEC
NON-INTERLACED
VIDEO BANDWIDTH = 4 MHz

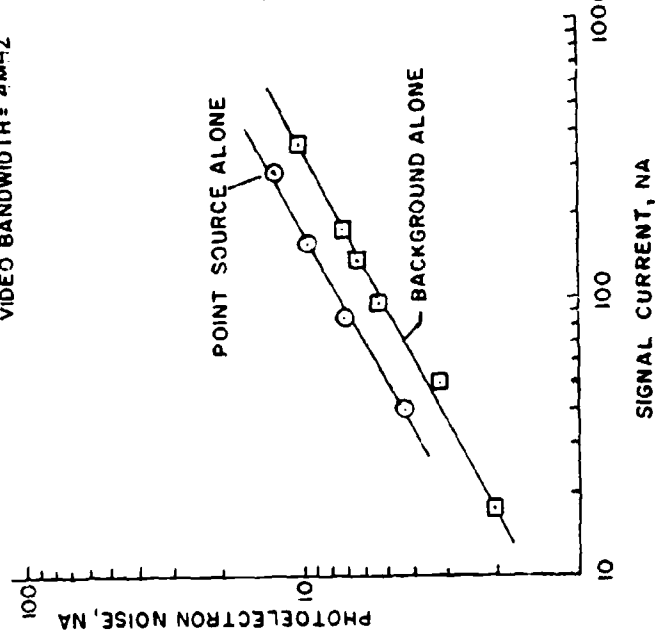


FIG. 3 PHOTOELECTRON NOISE, POINT SOURCE & UNIFORM BACKGROUND

RCA TYPE 4826 SIT TUBE
TARGET GAIN = 700
525 SCAN LINES
30 FRAMES/SEC
NON-INTERLACED
VIDEO BANDWIDTH = 4 MHz
FIXED BACKGROUND
CURRENT = 108 NA

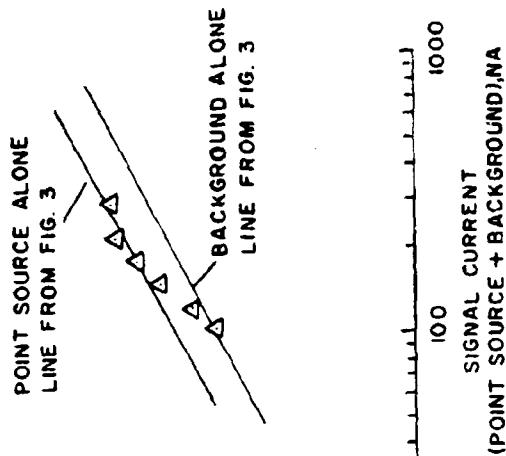


FIG. 4 PHOTOELECTRON NOISE, VARIABLE POINT SOURCE ON A FIXED BACKGROUND

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effective continuous scan in the y direction. Since the larger format (25 mm) Westinghouse tube requires 700 scan lines approximately 40 scan lines per mm of raster height is required in each instance.

It is noted that the noise current at the peak of a point source is larger than for a uniform background of equal signal current. It is easily shown that the expected noise currents should differ by $(a_s/a_n)^{1/2}$. For a Gaussian impulse response, $(a_s/a_n)^{1/2} = \sqrt{2}$. Reference to Figure 3 shows that the experimental noise measurements do indeed differ by $\sqrt{2}$.

Figure 4 is a plot of photoelectron noise measured at the peak of a point source immersed in a constant uniform background of 108 nanoamperes. The noise is plotted as a function of total signal current (point source current plus background current). At large signal currents when the point source current is large compared to the background, the data approach the point source alone data of Figure 3. The single point at 108 na corresponding to zero point source current falls on the curve of background alone in Figure 3. Thus for any combination of point source and background current, the measured noise current will be on or between the two straight lines of Figure 3.

SUMMARY AND CONCLUSIONS

The effect of a finite aperture on photoelectron noise has been described in some detail. Two equivalent sampling areas or apertures, a_s and a_n , were defined which permit a rapid calculation of signal-to-noise ratio when viewing a uniform scene, a point source immersed in a uniform scene, or a periodic pattern. The averaging areas may be visualized as fictitious areas on the photocathode over which photoelectrons are effectively averaged by the imaging device. The necessity for two sampling areas was demonstrated by calculating the signal-to-noise ratio for a point source in a uniform background. When considering background alone, Schade's N_e parameter is sufficient and was shown to be related to the averaging areas by $N_e^{-2} = (a_s^2/a_n)$ = a^* .

Data from two intensifier silicon type television tubes shows that a_s and a_n can be calculated from measured MTF data and the assumption of continuous sampling in the y direction. The latter assumption is valid for scan line densities equal to or greater than about 40 scan lines per mm of raster height.

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